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EXPANSION OF GAS CAVITY IN BRITTLE ROCK WITH A VIEW TO DILATATION PROPERTIES OF SOIL

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The experimental data of [1, 2] on blasts in rocks indicate that the mass velocity of the rock behind the front of the shock wave, comminuted by that shock wave, is described by the relation

$$v \sim r^{-n}, \quad n = 1.5 - 1.8.$$

The explanation of such a relationship may be connected with the effect of dilatation in the comminuted rock, the effect consisting of the dependence of the specific volume on the plastic shear deformations [3].

The equation of continuity and the relationships imposing kinematic limitations on the velocity components [1, 3]

$$d\rho/\rho dt + \operatorname{div} v = 0, \qquad (1)$$
$$I_1 - 2\Lambda\sqrt{I_2} = 0,$$

form a closed system for determining the velocity and density of the soil behind the front of the shock wave. Here ρ, v, I_1, and $I_2^{1/2}$ are the density, velocity, first and second invariants (deviator part) of the tensor of the deformation rate, and Λ is the dilatation rate.

The solution of the system of equations (1) in the spherical-symmetrical case with Λ = const leads to the following dependence of the velocity and density on the coordinates and the time:

$$v(r, t) = \lambda(t)/r^n,$$
$$\rho(r, t) = \rho^-(r_0)(r_0/r)^{2-n}, \; n = (2 - \Lambda)/(1 + \Lambda), \qquad (2)$$

where r, r_0 are the running and initial coordinates, respectively, of the particle; $\rho^-(r_0)$ is the density of the material at point r_0 at the instant the shock wave passes through that point; $\lambda(t) = a^n\dot{a} = v(R)R^n$; a and R are the radii of the cavity and of the front of the shock wave at the instant of time t; v(R) is the mass velocity of the particles behind the front of the shock wave

$$v(R) = \varepsilon(R)\dot{R}, \quad \varepsilon(R) = \frac{\rho_0^-(R) - \rho_0^+}{\rho_0^-(R)};$$

ρ_0^+ is the density of the soil after arrival of the shock wave.

We assume that the soil behind the shock wave is a plastic medium obeying the Mises–Schleicher condition

$$\sigma_r - \sigma_\theta = k + m(\sigma_r + 2\sigma_\theta).$$

The equation of motion in Lagrange variables can be written in the form

$$r_0^2 r^{-2} \rho_0^+ \frac{\partial v}{\partial t} = r^{-\alpha} \frac{\partial}{\partial r_0}\left[r^\alpha\left(\sigma_r(r) + \frac{k}{3m}\right)\right], \quad \alpha = \frac{6m}{2m+1}. \qquad (3)$$

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The correlation between the initial and the running coordinate can be obtained from the relationship

$$r^{n+1}-r_0^{n+1}=\int_{r_0}^{R}\frac{\rho_0^-(R')-\rho_0^+}{\rho_0^-(R')}\,dR'^{n+1}=a^{n+1}-a_0^{n+1}+\varepsilon\left(a_0^{n+1}-r_0^{n+1}\right). \tag{4}$$

Equations (2) and (4) make it possible to determine the degree of loosening of the comminuted soil behind the front of the shock wave

$$\frac{\rho(r)}{\rho_0^-}=\left[\frac{1-\left(\frac{a}{r}\right)^{n+1}}{1-\varepsilon}+\left(\frac{a_0}{r}\right)^{n+1}\right]^{\frac{2-n}{n+1}}.$$

To determine the dynamics of the development of the cavity we integrate Eq. (3), bearing in mind that the radial stress at the front of the shock wave $\sigma_r(R)$ can be expressed in terms of the radius and the velocity of the front of the shock wave or of the cavity

$$\sigma_r(R)=-\rho_0^+\varepsilon(R)\dot{R}^2-\sigma^*=-\rho_0^+\varepsilon^{-1}(R)\left(\frac{a}{R}\right)^{2n}\dot{a}^2-\sigma^*$$

(σ^* is the stress at which the phase of irreversible failure begins).

As a result we obtain

$$y'_{\bar{a}}+\frac{M(\bar{a})}{\bar{a}}y=\frac{b(\bar{a})}{\bar{a}}, \tag{5}$$

where

$$M(\bar{a})=\left[\frac{1}{2}\bar{a}^{n+1}F^{\alpha}_{2+n}(\bar{a})\right]^{-1}\left[n\bar{a}^{2n}F^{\alpha}_{3+2n}(\bar{a})+\varepsilon^{-1}x^{\alpha}\left(\frac{\bar{a}}{x}\right)^{2n}+n\bar{a}^{n+1}F^{\alpha}_{2+n}(\bar{a})\right];$$
$$b(\bar{a})=\left[\frac{1}{2}\bar{a}^{n+1}F^{\alpha}_{2+n}(\bar{a})\right]^{-1}\left\{\bar{a}^{\alpha}p(\bar{a})+\varkappa\left(x^{\alpha}-a^{\alpha}\right)-\frac{\sigma^*}{p_0}x^{\alpha}\right\}$$

and we introduce the following dimensionless values:

$$p(\bar{a})=\frac{\sigma_r(\bar{a})}{p_0},\quad \bar{a}=\frac{a}{a_i},\quad y=\frac{\rho_0^+}{p_0}\dot{a}^2,\quad \varkappa=\frac{k}{3mp_0},$$
$$x=\frac{R}{a_i},\quad \lambda=\bar{a}^n\sqrt{y},\quad \bar{a}^{n+1}-1=\varepsilon\left(x^{n+1}-1\right),$$
$$F_k^{\alpha}(\bar{a})=\int_1^x s^2r^{-(\alpha+k)}(s)\,ds,$$

where a_i and p_0 are the size of the cavity and the pressure determined by the initial stage in the development of the blast. To determine them, we use the following idea of the process of blast at this stage. At the initial instant a large amount of energy is liberated within a small volume, and this causes the rock to evaporate. The mass M of rock per kiloton of charge can be determined from experimental and theoretical data [4]. For instance, in granite 70 tons of rock per kiloton are evaporated. An estimate of the mass of the evaporated substance can be obtained by considering that in impulsive loading the evaporation of the material requires an energy approximately ten times greater than the bond energy of the substance [5] (for SiO_2 this value is 200 kcal/mole, which yields an estimate of $M \approx 65$ tons/kiloton). The radius of the evaporated zone and the initial pressure are given by the ratio $a_0^3 = 3MW/4\pi$, $p_0 = (\gamma - 1)\rho/M$.

The following phase of development of a powerful underground blast is determined by the hydrodynamic flow of the molten rock [1]. Taking the molten rock as an incompressible liquid, we can obtain the stress at the boundary of the melt. The phase of hydrodynamic motion ends when the stresses become equal to the theoretical strength of a single crystal $Y \approx 0.1E$. At that instant the phase of flow of the comminuted dilatating rock begins. The initial radius of this phase is determined by the relation $a_i = (p_0/Y)^{1/3\gamma}$, where γ is the exponent of the adiabatic curve of the vapors inside the cavity.

The initial velocity y_0 can be expressed in terms of the same characteristics by the relation $y_0 = [2/3(\gamma - 1)](Y/p_0)^{\gamma}$.

If we seek the final dimensions of the zones of failure, we obtain from Eq. (5) at the limit, when $\varepsilon x^{n+1} \gg 1$, the following expression for the volume V of the cavity at the instant of complete stopping:

for $n > 1 + \alpha$

$$V = W\left(\frac{10p_0}{E}\right)^{1/\gamma} \frac{M}{\rho_0^+}\left[\frac{2}{3(\gamma-1)}\left(\frac{E}{10p_0}\right)^{1/\gamma}\frac{p_0}{\sigma^*}\varepsilon^{\frac{\alpha}{n+1}}\frac{2n(n+1)}{2(n-1-\alpha)(2n-\alpha)}\right]^{\frac{3}{\mu}}, \quad \mu = 2n(n+1)/(2n-\alpha); \tag{6}$$

for $n < 1 + \alpha$

$$V = W\left(\frac{10p_0}{E}\right)^{1/\gamma} \frac{M}{\rho_0^+}\left[\frac{2(\alpha+1)}{3(\gamma-1)(\alpha+1-n)}\left(\frac{E}{10p_0}\right)^{1/\gamma}\frac{p_0}{\sigma^*}\varepsilon^{\frac{n-1}{n+1}}\right]^{3/\mu}, \quad \mu = 2(\alpha+1). \tag{7}$$

We use Eqs. (6) and (7) for estimating the regions of failure in the "Hardhat" underground blast [2] for which there are literature data on the characteristics of the soils and the initial pressure:

$$\alpha = 2,\ p_0 = 1.3\ \text{Mbar},\ \sigma^* = 1.3\cdot 10^{-3}\ \text{Mbar},\ \varepsilon^* = \frac{\sigma^*}{\rho_0^+ c^2},$$
$$E = 0.62\ \text{Mbar},\ n = 18,\ \frac{3}{4\pi}\frac{M}{\rho_0^+} = 6.3\ \text{m}^3/\text{kiloton}.$$

If we substitute these values into Eq. (7), we obtain a = 19.6 m. The radius of the zone of comminution R_c and the radius of jointing R_{jo} are obtained by estimates based on a quasistatic approach to the problem of expansion of the cavity [1]:

$$R_c = a\left(\frac{E}{\sigma^*(n+1)}\right)^{1/(n+1)} = 125\ \text{m},$$
$$R_{jo} = R_c\left(\frac{\sigma^*}{2\sigma_0}\right)^{1/2} = 376\ \text{m},$$

where σ_0 is the tensile strength of the soil. For the "Hardhat" experiment we may take σ_0 = 70 bar.

The estimates thus obtained agree well with the experimental data on the "Hardhat" blast [2]. If the role of dilatation is disregarded, the volume of the cavity comes out larger by a factor of 1.3 with n = 1.5 and by a factor of 1.06 with n = 1.8.

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A METHOD FOR INVESTIGATING MATERIAL PROPERTIES DURING DYNAMIC ELONGATION

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In the high-speed impact of flat plates tensile stresses arising as a result of the interaction of incident waves and waves reflected from contact and free surfaces can lead to scab fracture. The direct experimental measurement of the critical stress and the corresponding strain in the scabbing zone is very difficult, and therefore these quantities are estimated by starting from various indirect measurements and making certain assumptions about the behavior of the material under elongation at a high strain rate. This problem is treated in a number of papers [1-8] in which scab fractures are studied experimentally and analytically. A detailed bibliography of this problem is given in [1, 7, 8]. These papers are characterized by the use of an elastic model of the material in determining the scab stress. We present an experimental—theoretical method for determining the stress—strain curve of a material clear up to scabbing using the more complicated elastoplastic model of a solid.

The basic equations and relations describing the one-dimensional unsteady motion of a continuous medium are written in the Lagrange—Euler form and are similar to those used in [9, 10].

The equation of motion

$$(1/V)\rho_0 du/dt = \partial\sigma/\partial x, \tag{1}$$

where ρ_0 is the initial density; u is the mass velocity; $-p + s = \sigma$ is the total stress; s is the shear part of the total stress; p is the bulk pressure (the spherical part of the stress tensor); and $\rho_0/\rho = V$ is the compressive strain.

The equation of continuity

$$(1/V)dV/dt = \partial u/\partial x. \tag{2}$$

The energy equation

$$dE/dt - Vsd\varepsilon/dt + pdV/dt = 0, \tag{3}$$

where $\varepsilon = \rho_0/\rho - 1$.

The equation of the stress deviator

$$ds/dt = 2\mu(d\varepsilon/dt - (1/3V)dV/dt), \tag{4}$$

where μ is the Lamé constant.

The rate of strain

$$d\varepsilon/dt = \partial u/\partial x. \tag{5}$$

The equation of state in the Mie—Grüneisen form

$$p_1 = p_H + \rho\Gamma(E - E_H), \tag{6}$$

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